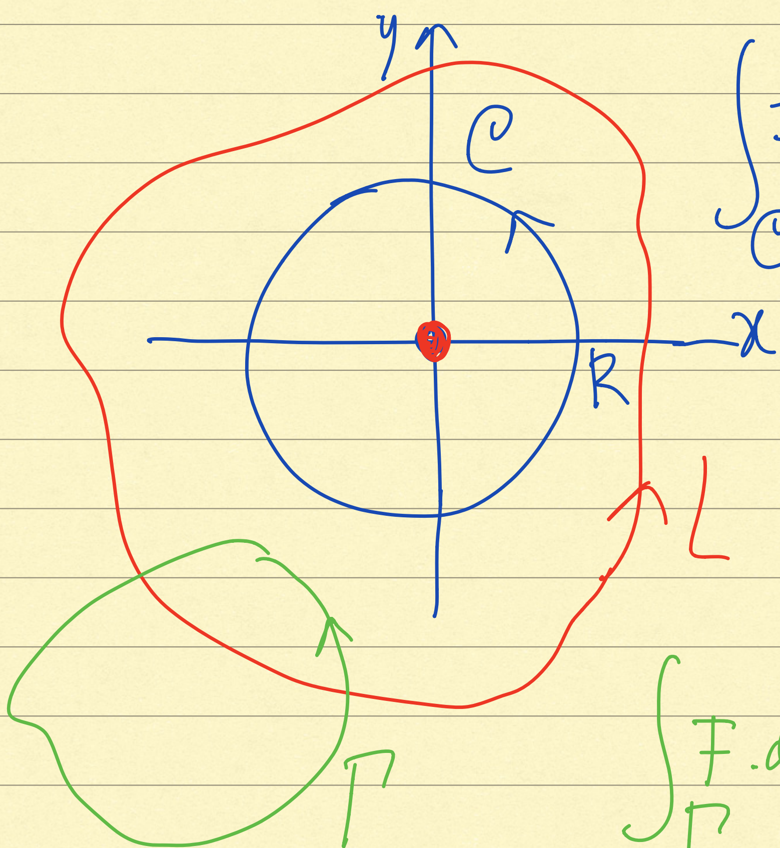
Teorema de GREEN em $\mathbb{R}^2$

$$F(x,y) = \left(-\frac{y}{x^2+y^2} , \frac{x}{x^2+y^2} \right)$$

$$(x,y) \neq (0,0)$$

$\boxed{F \text{ é fechado}}$

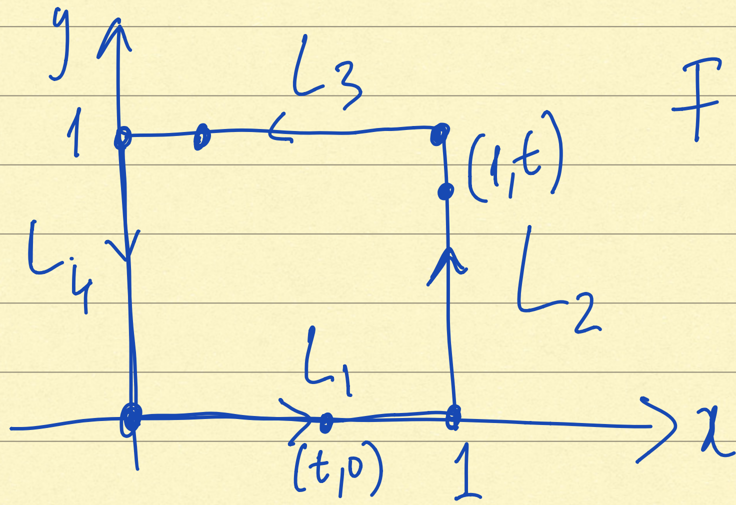
$C = \{ (x,y) \in \mathbb{R}^2 : x^2+y^2 = R^2 \}$ ↺ $R>0$.



$$\int_C F \cdot dg = 2\pi, \ \forall_{R>0}$$

$$\int_L F \cdot dg = ?$$

$$\int_\Gamma F \cdot dg = ?$$



$F: \mathbb{R}^2 \to \mathbb{R}^2, C^1$

$L = L_1 \cup L_2 \cup L_3 \cup L_4$

$F(x,y) = \big(P(x,y), Q(x,y)\big)$

$L_1: \quad g(t) = (t,0) \quad ; \quad 0 \le t \le 1.$

$$g'(t) = (1,0)$$

$$\int_{L_1} F \cdot dg = \int_0^1 F(g(t)) \cdot g'(t)\, dt$$

$$= \int_0^1 F(t,0) \cdot (1,0)\, dt$$

$$= \int_0^1 \big(P(t,0), Q(t,0)\big) \cdot (1,0)\, dt$$

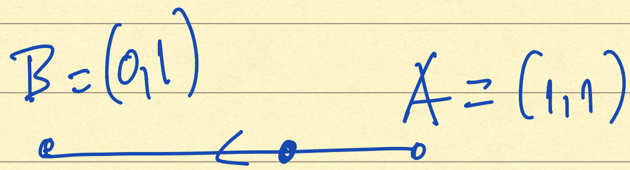
$$= \int_0^1 P(t,0)\, dt$$

$L_2: \quad g(t) = (1,t) \quad ; \quad 0 \le t \le 1$

$\qquad g'(t) = (0,1)$

$$\int_{L_2} F \cdot dg = \int_0^1 F(g(t)) \cdot g'(t)\, dt = \int_0^1 Q(1,t)\, dt$$

$L_3:$



$$g(t) = A + t(B - A), \quad 0 \le t \le 1$$

$$= (1,1) + t(-1,0) = (1-t, 1)$$

$g'(t) = (-1,0)$

$$\int_{L_3} F \cdot dg = -\int_0^1 P(1-t,1)\, dt \qquad 1-t=s$$

$$= -\int_0^1 P(s,1)\, ds = -\int_0^1 P(t,1)\, dt$$

L_4:

$A = (0,1)$

$g(t) = A + t(B - A)$; $0 \le t \le 1$

$B = (0,0)$

$= (0,1) + t(0,-1)$

$= (0, 1-t)$

$$g'(t) = (0,-1)$$

$$\int_{L_4} F \cdot dg = -\int_0^1 Q(0, 1-t)\,dt$$

$1 - t = 1$

$$= -\int_0^1 Q(0,t)\,dt$$

$$\int_L F \cdot dg = \int_0^1 P(t,0)\,dt + \int_0^1 Q(1,t)\,dt - \int_0^1 P(t,1)\,dt - \int_0^1 Q(0,t)\,dt$$

$$= - \int_0^1 \Big[-P(t,0) + P(t,1) \Big] \, dt \quad +$$

$$+ \int_0^1 \Big[Q(1,t) - Q(0,t) \Big] \, dt$$

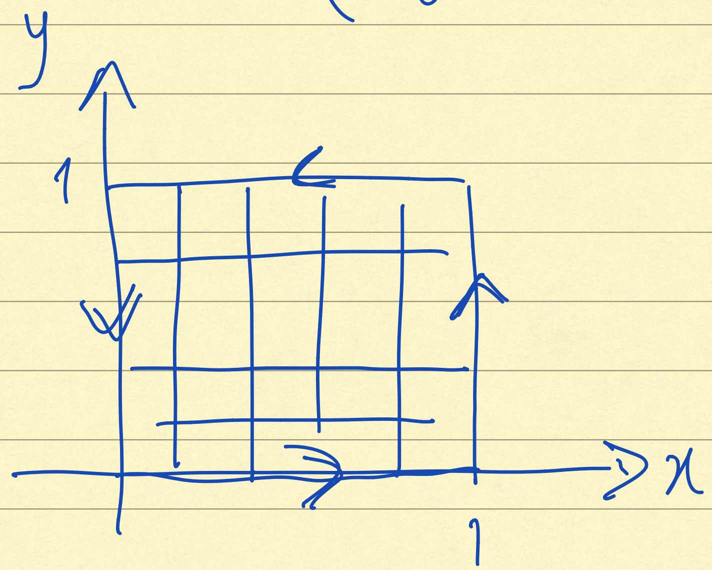
$$\boxed{\begin{array}{c} f \in C^1 \\ f(b) - f(a) = \int_a^b f'(t)\, dt \end{array}}$$

$$= - \int_0^1 \left[\int_0^1 \frac{\partial P}{\partial y}(\underset{x}{t}, y)\, dy \right] d\underset{x}{t} \quad +$$

$$+ \int_0^1 \left[\int_0^1 \frac{\partial Q}{\partial x}(x, \underset{y}{t})\, dx \right] d\underset{y}{t}$$

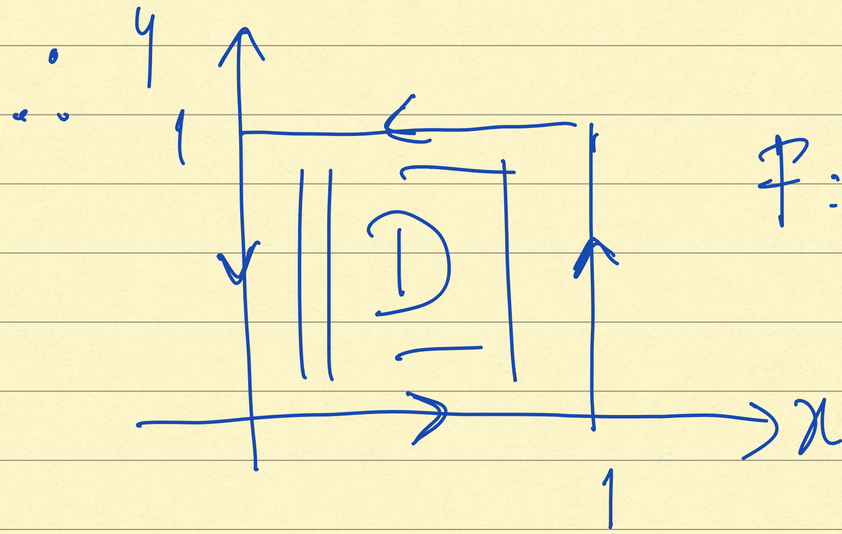
$$= \int_0^1 \left(\int_0^1 \frac{\partial Q}{\partial x}(x,y)\, dx \right) dy$$

$$- \int_0^1 \left(\int_0^1 \frac{\partial P}{\partial y}(x,y)\, dy \right) dx$$



$\boxed{\text{Fubini}}$

$$= \int_0^1 \left(\int_0^1 \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx \right) dy$$



$F: \mathbb{R}^2 \to \mathbb{R}^2$, C^1

$D = \{(x,y) \in \mathbb{R}^2 : 0 < x < 1 ; \ 0 < y < 1\}$ aberto, limitado

$\partial D = L_1 \cup L_2 \cup L_3 \cup L_4$

+ Teorema de Fubini

$$\Rightarrow \boxed{\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\,dy = \int_{\partial D} F \cdot dg}$$

Green!!!

$$F = (P, Q)$$

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy = \boxed{\oint_{\partial D} P\,dx + Q\,dy}$$

$$\int_L F \cdot dg = \int_a^b F(g(t)) \cdot g'(t)\,dt =$$

$$\int_a^b \left(P \frac{dx}{\cancel{dt}} + Q \frac{dy}{\cancel{dt}}\right) \cancel{dt}$$

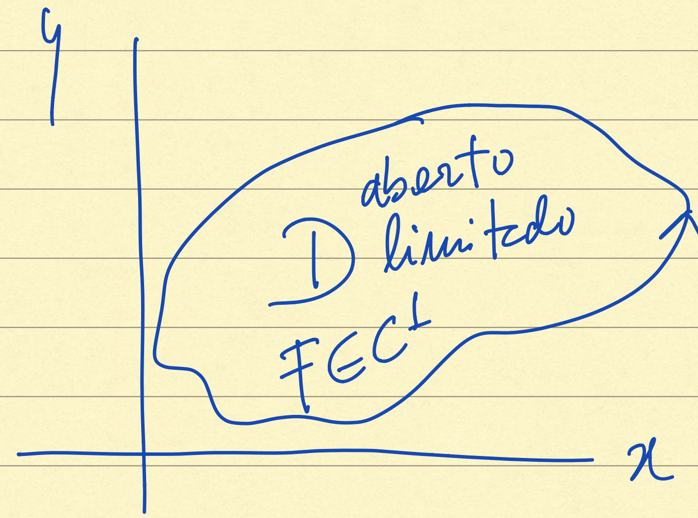
"heresia"

$$\int P\,dx + Q\,dy$$

$$g(t) = (x(t), y(t))$$

$$g'(t) = (x'(t), y'(t))$$

$$= \left(\frac{dx}{dt}(t), \frac{dy}{dt}(t)\right)$$



$F: \mathbb{R}^2 \to \mathbb{R}^2, C^1$

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy = \oint_{\partial D} P\,dx + Q\,dy$$

$$\oint_{\partial D} F \cdot dg$$

<u>interior</u>

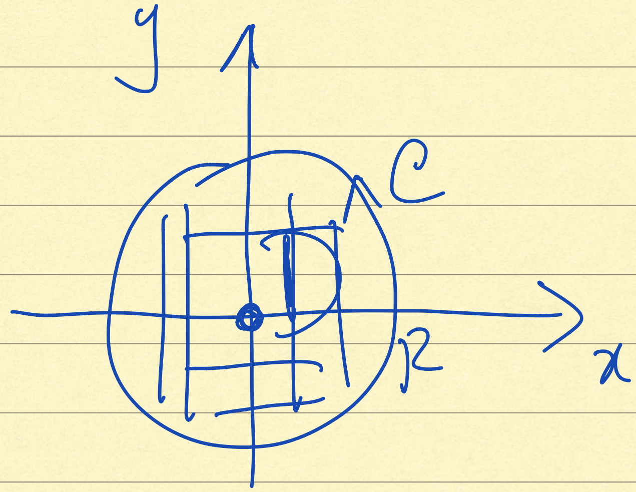
<u>fronteira</u>

1- Se $\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$ (<u>F é fechado</u>)

então $\oint_{\partial D} P\,dx + Q\,dy = 0$!

Exemplo:

$$F(x,y) = \left(-\frac{y}{x^2+y^2} , \frac{x}{x^2+y^2} \right) ; (x,y) \neq (0,0)$$



$$\oint_C F \cdot dg = 2\pi \neq 0$$

$$D = \{ (x,y) \in \mathbb{R}^2 : x^2+y^2 < R^2 \}$$

F não é C^1 em D.

T. de Green não se aplica !!!

Trupue: